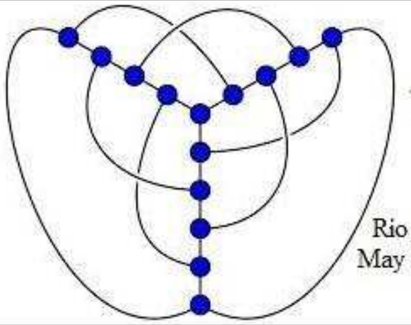

7th Crossing Number Workshop

Rio de Janeiro, Brazil
May 18-22, 2015

Crossing Number Workshop'2015

Book of Abstracts

Universidade do Estado do Rio de Janeiro

Rio de Janeiro, May 18–22, 2015

Crossing Number Workshop'2015

The *crossing number* of a graph $G = (V, E)$ is the minimum number of edge crossings in a drawing of G on the plane. The crossing number of a graph has many variants, among them the *rectilinear* crossing number having the edges of E represented by straight lines, and the crossing number in $k \geq 1$ *pages*, where the vertices of V are aligned in a straight line called *spine* and the edges of E partitioned into a collection of k distinct semi-planes whose pairwise intersection is the spine.

Since six years ago, a representative group of crossing number researchers has organized focused workshops. The Crossing Number Workshops have gathered over the world, many of the reseachers working with topologic and algorithmic features on crossing numbers. This initiative has substantively contributed for the development of many results in crossing number theory.

And it is for our joy that this time, the city of Rio de Janeiro has been chosen to host from May 18 to May 22, 2015 – The Seventh Crossing Number Workshop.

The Crossing Number Workshop'2015 was organized with the support of CNPq, CAPES and FAPERJ. The State University of Rio de Janeiro offered the space for the realization of the plenaries and communications. The Local Organizing Committee was formed by Researchers of State University of Rio de Janeiro – UERJ, Federal University of Rio de Janeiro – UFRJ, Federal Fluminense University - UFF, and Federal University of the ABC Region - UFABC. Hoping the consequence of this organization has fruitful results, the Local Organizing Committee welcome all participants.

Organizing Committee

Leticia Rodrigues Bueno
Luerbio Faria
Celina Miraglia Herrera de Figueiredo
Sulamita Klein
Carlos Alberto de Jesus Martinhon
Fabiano de Souza Oliveira

Steering Committee

Drago Bokal
Markus Chimani
Petr Hliněný
Gelasio Salazar

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Abstracts

1 Joint Embedding ("Two Maps on One Surface"): Computational Complexity. Petr Hliněný

A joint embedding of two graphs G_1, G_2 on a surface Σ is a drawing of $G_1 + G_2$ such that the restriction of it to each one of G_1, G_2 is a cellular embedding on Σ. The task is find a joint embedding which minimizes the number of pairwise edge crossings between the edges of G_1 and the edges of G_2. We discuss why this problem is computationally hard, which in turn means that there are likely no "nice" theoretical descriptions of the optimal solution.

2 Pseudolinear drawings. Gelasio Salazar

We recently answered a few open questions, posed by Marcus Schaeffer, on the complexity of the pseudolinear crossing number. We will give a brief outline of these results. This is joint work with Jess Leaos and César Hernndez-Vélez.

3 Crossing number and highly connected graphs. Alan Arroyo

In a recent work with Bruce Richter we found a "simple" graph-theoretical characterization of graphs having crossing number at least 2: A 4-connected non-planar graph has crossing number at least 2 if and only if for every pair of disjoint edges there are vertex disjoint cycles containing these edges. It would be nice to explore if there are analogous results for graphs with crossing number at least k, with $k > 2$. For example, for $k = 3$, it is true that graphs with high connectivity and crossing number at least 3 can be characterized in terms of a combinatorial property?

4 Crossing-critical edges and Kuratowski subraphs. César Hernández-Vélez

We call the graphs K_5 and $K_{3,3}$ of Kuratowski graphs. An edge e of a graph G is a *Kuratowski edge* if e belongs a subgraph H of G homeomorphic to a Kuratowski graph. We recall that an edge e is crossing-critical if the crossing number of $G - e$ is less than the crossing number of G. We present some relationships between crossing-critical edges and Kuratowski edges. Joint work with Drago Bokal and Jesús Leaños.

5 On the optimal drawings of $K_{5,n}$. Carolina Medina

We will outline the proof of a recent result that classifies all the crossing-minimal drawings of $K_{5,n}$. In a nutshell, we prove that besides the classical Zarankiewicz drawing there is only one other way to draw $K_{5,n}$ with the minimum number of crossings, for any positive even integer n. This is one of the handful of results around Zarankiewicz's conjecture that does not rely on the use of computers. This is joint work with Cesar Hernandez-Velez and Gelasio Salazar.

6 The Crossing Number of $K_{5,n}$ on The Projective Plane. André Carvalho Silva

It has been long conjectured by Zarankiewicz (1954) that the planar crossing number of the complete bipartite graph $K_{m,n}$ equals to $Z(m,n) = \frac{n}{2}\frac{n-1}{2}\frac{m}{2}\frac{m-1}{2}$. The conjecture is still open and it seems difficult.

For other surfaces, no similar conjecture is known. Richter and Siran (1996) determined the crossing number of $K_{3,n}$ for all surfaces. They also presented an upper bound for the crossing number of $K_{m,n}$. Later Pak (2003 and 2008) proved that this value is optimal for the crossing number of $K_{4,n}$ on projective plane and the torus. However, this is not true for $K_{5,n}$ on the projective plane.

In this talk we consider the problem of determining the crossing number of $K_{5,n}$ on the projective plane. More specifically, we present an improved upper bound for the crossing number of $K_{5,n}$ on the projective plane and we discuss the problem of finding better lower bounds.

7 Automatic Crossing Number Proofs. Markus Chimani

There are integer linear programming approaches that allow to compute the optimum crossing numbers for given (small/sparse) graphs. However, their implementations are so intricate, involved, and extensive that formally checking them is close to impossible. Hence, doubts about the algorithms' correctness may always remain. We discuss how to extract independently checkable proofs from these approaches. These proofs, while still machine dependent, only require a minimal set of easily checkable subprograms.

8 Crossing graphs of book drawings are circular graphs. Drago Bokal

Let G be a graph and Σ_k a surface, obtained from k disjoint semi-planes (referred to as pages of Σ_k) identified at their boundary line (referred to as the spine of Σ_k). A book or a k-page drawing of a graph G is a drawing of G in Σ_k, such that the vertices of G are drawn in the spine of Σ_k and the edges are drawn in the open pages of Σ_k (k-page crossing number) of G is the smallest number of crossings of some k-page drawing of G. We denote it by $crn_k(G)$. The page number $p(G)$ of G is the smallest k, such that G can be drawn in Σ_k with no crossings.

Let G be a graph and π a circular ordering of vertices of G. The crossing graph of G under π is the graph $C(G,\pi)$, whose vertices are edges of G, two of them, uv and xy, being adjacent if either $uxvy$ or $uyvx$ is a subsequence of either π or π^{-1}.

Buchheim and Zheng observed that $2-$page crossing number of G is equal to the smallest $|E(C(G,\pi))| - mc(C(G,\pi))$, where $mc(G)$ denotes the size of the largest $2-$cut of $C(G,\pi)$ and the minimum is taken over all circular orderings π of $V(G)$. This observation has several extensions:

Theorem: Let G be a graph and $C(G,\pi)$ its crossing graph. Then the following hold:

- $p(G) = \min_\pi \chi(C(G,\pi))$, ie. the page number of G is equal to the smallest chromatic number of some crossing graph of G,
- $crn_k(G) = |E(C(G,\pi))| - mc_k(C(G,\pi))$, ie. the $k-$page crossing number of G is equal to the smallest number of edges of some crossing graph of G that are not in some $k-$cut of G.

In this context, Bokal, Kotrbčík, and Repolusk observed that the class of crossing graphs of graphs is precisely the class of circle graphs, i.e. intersection graphs of chords of a circle. Among other things, they showed that this class properly contains all K_4-minor free graphs.

Problem: Can we use knowledge on circle graphs to extend our knowledge on book embeddings?

9 Computational tools for the rectilinear crossing number and other geometric problems. Ruy Fabila

I will detail implementations that we have developed in the last couple of years. This programs were made as tools for various problems in Combinatorial Geometry. I will talk about the current status of the project as well as many possible applications"

10 From MultiCut to Crossing Number. Rafael Pocai

Sergio Cabello, in 2013, presented a reduction from MultiwayCut with 3 terminals to Crossing Number, showing that there is a constant c such that there is no c-approximation for the second problem. We present a reduction from MultiCut to Crossing Number, restricted to instances with at most 9 pairs of terminals. This reduction gives no new lower bound for approximation, but since known approximation algorithms for MultiCut are worse than for MultiwayCut, new bounds may arrive from it.

11 Intersection graphs of geometric figures between two parallel lines. Fabiano Oliveira

A graph G is called an *intersection graph* if there is a family of sets $\{F_v : v \in V(G)\}$ such that, for all distinct $u, v \in V(G)$, $F_v \cap F_u \neq \emptyset \iff uv \in E(G)$. Such a family is called a model of G. Although any graph is the intersection graph of some family of sets, interesting graph classes arise when restriction on the models are imposed. In talk, we shall present the classical intersection graphs of models whose elements consist of geometric figures (curve and straight lines, trapezoids, triangles, etc.) delimited by two parallel lines and discuss the problem of recognizing such graphs. Moreover, it will be noted that the crossing number problem could be generalized to work with such geometrical kind of models.

12 Representing Planar Graphs Using Intersecting Curves. Martin Derka

String graphs are graphs that can be represented using curves in the plane as follows: Given a graph $G = (V, E)$, every vertex $v \in V$ is assigned a curve $\mathbf{v}$ so that curves $\mathbf{u}, \mathbf{v}$ intersect if and only if G contains an edge $(u, v) \in E$. We present some recent results in this field. Namely, we show that every planar graph is a string graph in which all curves are paths in an orthogonal grid, no two curves intersect more than once, and every curve has at most two bends. Furthermore, we show that planar graphs with tree-width at most 3 do not require more than one bend per curve, and restrict the required shapes of curves even further for outer-planar graphs, Halin graphs and IO graphs.

13 Book-crossing numbers of circulant graphs. Paul Kainen

A few special cases will be described and a variety of open problems will be considered. We also look at the relation with book thickness when the pages are constrained to be (crossing-free) matchings.

14 Crossing numbers of Complete graphs. Daniel J. McQuillan

Hill's conjecture for the crossing number of the complete graph has been verified up to n=12. After briefly describing a standard approach, including its reliance on computers, we outline an alternative. Our approach involves the interplay between optimal drawings and drawings that are far from optimal. Our general results are enough to provide a complete proof that the crossing number of K_9 is 36. This is joint work with R. Bruce Richter.

15 Join products and crossing numbers of derived graphs. Marián Klešč

The *crossing number* $cr(G)$ of a graph G is the minimum possible number of edge crossings in a drawing of G in the plane. The investigation on the crossing numbers of graphs is a classical and however very difficult problem. The problem of reducing the number of crossings was therefore not only studied by the graph theory community, but also by VLSI communities and computer scientists. Garey and Johnson have proved that the problem to determine the crossing number of a graph is NP-complete. The crossing numbers of some classes of graphs have been obtained. It was shown by D.J. Kleitman that the crossing number of the complete bipartite graph $K_{m,n}$ is $\lfloor\frac{m}{2}\rfloor\lfloor\frac{m-1}{2}\rfloor\lfloor\frac{n}{2}\rfloor\lfloor\frac{n-1}{2}\rfloor$ for all $m \leq 6$ and all n. Let G and H be two disjoint graphs. The *join product* of G and H, denoted by $G + H$, is obtained from vertex–disjoint copies of G and H by adding all possible edges between $V(G)$ and $V(H)$. For $|V(G)| = m$ and $|V(H)| = n$, the edge set of $G + H$ is the union of disjoint edge sets of the graphs G, H, and the complete bipartite graph $K_{m,n}$. The Kleitman's result enables us to establish the crossing numbers of several join products. In the talk, we summarize the known results concerning crossing numbers for the join products of two graphs. We present several ideas how to obtain the exact values of crossing numbers for the Cartesian products of special graphs with stars and trees. Besides, several open problems concerning this topic will be presented.

16 Bounds on the crossing number of the n-cube. Luerbio Faria

In this talk we describe some results in the literature about upper bounds for the crossing number, rectilinear and k-page crossing number on the n-cube graph. This is a joint work with Imrich V'rto, Celina Miraglia Herrera de Figueiredo and Bruce Richter.

17 Monotone and c-monotone drawings of the complete graph. Pedro Ramos

Assume that vertices $1, 2, \ldots, n$ are on a line ℓ. A monotone drawing is a drawing in which edges are curves monotone wrt to ℓ. Changing the line by a circle on the cylinder we get c-monotone drawings. In this talk we review recent results and open problems about monotone and c-monotone drawings of the complete graph.

18 Deciding monotonicity of good drawings of the complete graph. Thomas Hackl

We provide an $O(n^5)$ time algorithm for deciding whether for a complete graph K_n, given in terms of its rotation system, there exists a good drawing using only x-monotone arcs.

19 All good drawings of small complete graphs. Birgit Vogtenhuber

Good drawings, also known as simple topological graphs, are drawings of graphs such that any two edges intersect at most once. We are in particular interested in good drawings of the complete graph in connection with the crossing number. We describe our techniques for generating all different weak isomorphism classes (rotation systems) of good drawings of the complete graph for up to nine vertices. As an application of the obtained data we present results on the crossing number of these drawings.

20 Research Problems on good drawings of the complete graph. Oswin Aichholzer

We will discuss open problems and research questions about the relation of rotation systems, good drawings, and the crossing number of complete graphs.

21 Crossings in cylindrical drawings of the complete graph. Silvia Fernandez

A drawing of the complete graph on the plane is called *cylindrical* if there are two vertex-disjoint cycles covering all the vertices of the original graph and whose edges are crossing free. In the late 50s Hill found cylindrical drawings of K_n with few crossings. The actual number of crossings has been denoted by $Z(n)$. These drawings were conjectured to achieve the minimum number of crossings over all drawings of K_n. This is known as the Harary-Hill conjecture. Recently, it was proved that this conjecture holds when the minimum is restricted to *shellable* drawings of K_n. Because cylindrical drawings of K_n are shellable, they all have at least $Z(n)$ crossings. However, except for small values of n, the only cylindrical drawings known to achieve this minimum are Hill's drawings. The question for this talk is, are there any others?

22 Around Albertson's Conjecture. Marek Dernar

Bruce Richter proposed the following problem during the Guanajuato Workshop on Crossing numbers in February of 2013. Suppose $r \geq 5$ and $cr(G) \geq cr(K_r)$. Does it follow that $cr(CG) \geq cr(K_{r+1})$? This question was motivated by Albertson's conjecture: If the chromatic number $\chi(G)$ is r, then $cr(G) \geq cr(K_r)$. Richter's problem can be proven in the case $r = 5$, and disproved when $r = 6$. A natural variation of Richter's problem is the following. For each integer $k \geq 1$, let $f(k)$ be the smallest crossing number of the cone CG of a graph G with crossing number at least k. We obtain some exact values and lower and upper bound of this function.

23 On the Crossing Lemma. Bernardo Abrego

The Crossing Lemma due to Ajtai, Chvátal, Newborn, and Szemerédi states that the crossing number of a graph, with n vertices and m edges, drawn in the plane is at least $\Omega(m^3/n^2 - n)$. This inequality is asymptotically sharp except for the multiplicative constant. In this talk, we survey these results and we discuss some possible plans for future improvements.